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Irreducible antifield-BRST approach to reducible gauge theories

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Abstract. An irreducible antifield BRST quantization method for reducible gauge theories is proposed. The general formalism is illustrated in the case of the Freedman–Townsend model.

1. Introduction

There are two main approaches to the quantization of gauge theories with open algebras, both related to the BRST symmetry. The first one is based on the Hamiltonian formalism [1–6], while the second one relies on the Lagrangian formulation [6–11]. Both methods can be applied to irreducible, as well as to reducible gauge theories. In the irreducible case the ghosts can be regarded as one-forms dual to the vector fields associated with the gauge transformations. In the reducible situation this interpretation fails, so it is necessary to add ghosts of ghosts together with their antifields. The ghosts of ghosts are required in order to accommodate the reducibility relations to the cohomology of the (a model of) longitudinal exterior differential along the gauge orbits [6], while their corresponding antifields ensure the acyclicity of the Koszul–Tate operator at non-vanishing antighost numbers.

In this paper we propose an irreducible BRST approach to the quantization of on-shell reducible Lagrangian gauge theories. In consequence, the ghosts of ghosts and their antifields are absent. Our treatment mainly focuses on: (i) transforming the initial redundant gauge theory into an irreducible one in a manner that allows the substitution of the BRST quantization of the reducible system with that of the irreducible theory, and (ii) quantizing the irreducible theory along the antifield-BRST ideas. We mention that the idea of replacing a reducible system by an equivalent irreducible one appeared for the first time in the Hamiltonian context [6, 12] and was developed recently in the case of the quantization of Hamiltonian systems with off-shell reducible first-class constraints [13].

Our paper is structured in five sections. In section 2 we start with an L-stage reducible theory, and derive an irreducible system by means of constructing an irreducible Koszul–Tate differential associated with the original reducible one. The irreducible Koszul–Tate complex is obtained by requiring that all the antighost number two co-cycles become trivial under an appropriate redefinition of the antighost number two antifields. This request implies the enlargement of both field and antifield spectra. Section 3 focuses on the derivation of the irreducible BRST symmetry corresponding to the irreducible theory inferred in section 2, emphasizing that we can replace the antifield BRST quantization of the reducible theory by

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that of the irreducible system. In section 4 we illustrate our procedure in the case of the Freedman–Townsend model. Section 5 ends the paper with some conclusions.

2. Derivation of the irreducible theory

2.1. The problem

Our starting point is the gauge invariant Lagrangian action

$$S_0[\Phi^{\alpha_0}] = \int \mathrm{d}^D x \, \mathcal{L}_0(\Phi^{\alpha_0}, \partial_\mu \Phi^{\alpha_0}, \ldots, \partial_{\mu_1} \ldots \partial_{\mu_l} \Phi^{\alpha_0}) \tag{1}$$

subject to the gauge transformations

$$\delta_\epsilon \Phi^{\alpha_0} = Z^{\alpha_0}_{\ \ \alpha_1} \epsilon^{\alpha_1} \qquad \alpha_0 = 1, \ldots, M_0 \qquad \alpha_1 = 1, \ldots, M_1 \tag{2}$$

which are assumed to be L-stage reducible

$$Z^{\alpha_0}_{\ \ \alpha_1} Z^{\alpha_1}_{\ \ \alpha_2} = C^{\alpha_0 \beta_0}_{\alpha_2} \frac{\delta S_0}{\delta \Phi^{\beta_0}} \qquad \alpha_2 = 1, \ldots, M_2, \tag{3}$$

$$Z^{\alpha_1}_{\ \ \alpha_2} Z^{\alpha_2}_{\ \ \alpha_3} = C^{\alpha_1 \beta_0}_{\alpha_3} \frac{\delta S_0}{\delta \Phi^{\beta_0}} \qquad \alpha_3 = 1, \ldots, M_3, \tag{4}$$

$$\vdots$$

$$Z^{\alpha_{L-1}}_{\ \ \alpha_L} Z^{\alpha_L}_{\ \ \alpha_{L+1}} = C^{\alpha_{L-1} \beta_0}_{\alpha_{L+1}} \frac{\delta S_0}{\delta \Phi^{\beta_0}} \qquad \alpha_{L+1} = 1, \ldots, M_{L+1} \tag{5}$$

where $\delta S_0 / \delta \Phi^{\beta_0} = 0$ stand for the field equations. For the sake of notational simplicity we take the fields to be bosonic. The subsequent discussion can be straightforwardly extended to fermions modulo the introduction of some appropriate phase factors.

The reducible BRST symmetry corresponding to the above reducible theory, $s_R = \delta_R + \sigma_R + \cdots$, contains two basic differentials. The first one, δ_R, named the Koszul–Tate differential, realizes an homological resolution of smooth functions defined on the stationary surface of field equations, while the second one, σ_R, represents a model of longitudinal derivative along the gauge orbits and accounts for the gauge invariances. For first-stage reducible theories, the construction of δ_R requires the introduction of the antifields $\Phi^*_{\alpha_0}$, $\eta^*_{\alpha_1}$ and $C^*_{\alpha_2}$, with the Grassmann parities (ε) and antighost numbers (antigh) given by

$$\varepsilon(\Phi^*_{\alpha_0}) = 1 \qquad \varepsilon(\eta^*_{\alpha_1}) = 0 \qquad \varepsilon(C^*_{\alpha_2}) = 1 \tag{6}$$

$$\mathrm{antigh}(\Phi^*_{\alpha_0}) = 1 \qquad \mathrm{antigh}(\eta^*_{\alpha_1}) = 2 \qquad \mathrm{antigh}(C^*_{\alpha_2}) = 3. \tag{7}$$

The standard definitions of δ_R are

$$\delta_R \Phi^{\alpha_0} = 0 \qquad \delta_R \Phi^*_{\alpha_0} = -\frac{\delta S_0}{\delta \Phi^{\alpha_0}} \tag{8}$$

$$\delta_R \eta^*_{\alpha_1} = Z^{\alpha_0}_{\ \ \alpha_1} \Phi^*_{\alpha_0} \tag{9}$$

$$\delta_R C^*_{\alpha_2} = -Z^{\alpha_1}_{\ \ \alpha_2} \eta^*_{\alpha_1} - \tfrac{1}{2} C^{\alpha_0 \beta_0}_{\alpha_2} \Phi^*_{\alpha_0} \Phi^*_{\beta_0}. \tag{10}$$

The antifields $C^*_{\alpha_2}$ are necessary in order to kill the antighost number two non-trivial co-cycles

$$\nu_{\alpha_2} = Z^{\alpha_1}_{\ \ \alpha_2} \eta^*_{\alpha_1} + \tfrac{1}{2} C^{\alpha_0 \beta_0}_{\alpha_2} \Phi^*_{\alpha_0} \Phi^*_{\beta_0} \tag{11}$$

resulting from (9) via the reducibility relations (3). In the case of two-stage reducible theories, apart from the above antifield spectrum, one should add the antifields $C^*_{\alpha_3}$, with $\varepsilon(C^*_{\alpha_3}) = 0$, $\mathrm{antigh}(C^*_{\alpha_3}) = 4$, in order to kill the existing antighost number three co-cycles yielded by (10) if one takes into account the reducibility equations. In general, for an L-stage reducible system the antifield spectrum will contain the variables $\Phi^*_{\alpha_0}$, $\eta^*_{\alpha_1}$ and $(C^*_{\alpha_k})_{k=2,\ldots,L+1}$, where

$\varepsilon(C^*_{\alpha_k}) = k + 1 \bmod 2$, $\mathrm{antigh}(C^*_{\alpha_k}) = k + 1$, that are introduced in order to prevent the appearance of any non-trivial co-cycle at positive antighost numbers.

The problem to be investigated in this section is the derivation of an irreducible theory associated with a starting L-stage reducible gauge system. In this light, our main idea is to redefine the antifields $\eta^*_{\alpha_1}$ in such a way that all the non-trivial co-cycles (11) become trivial. The triviality of these co-cycles further implies that the antifields $(C^*_{\alpha_k})_{k=2,\ldots,L+1}$ are no longer necessary as there are also no non-trivial co-cycles at antighost numbers greater that two. The implementation of this idea leads to an irreducible gauge theory that possesses the same physical observables as the original reducible one. In order to clarify our irreducible mechanism, we gradually investigate the cases $L = 1, 2$, and then generalize the results to an arbitrary L.

2.2. The case $L = 1$

Here we start with equations (8)–(10) and the reducibility relations (3). In the light of the idea exposed above, we redefine the antifields $\eta^*_{\alpha_1}$ as

$$\eta^*_{\alpha_1} \rightarrow \tilde{\eta}^*_{\alpha_1} = \eta^*_{\alpha_1} - Z^{\beta_1}{}_{\beta_2} \bar{D}^{\beta_2}_{\alpha_2} A_{\alpha_1}{}^{\alpha_2} \eta^*_{\beta_1} - \tfrac{1}{2} C^{\alpha_0\beta_0}_{\beta_2} \bar{D}^{\beta_2}_{\alpha_2} A_{\alpha_1}{}^{\alpha_2} \Phi^*_{\alpha_0} \Phi^*_{\beta_0} \tag{12}$$

where $\bar{D}^{\beta_2}_{\alpha_2}$ is the inverse of $D^{\beta_2}_{\alpha_2} = Z^{\alpha_1}{}_{\alpha_2} A_{\alpha_1}{}^{\beta_2}$ and $A_{\alpha_1}{}^{\beta_2}$ are some functions that may involve the fields Φ^{α_0}, taken such that $\mathrm{rank}(D^{\beta_2}_{\alpha_2}) = M_2$. The next step is to replace (9) with

$$\delta\tilde{\eta}^*_{\alpha_1} = Z^{\alpha_0}{}_{\alpha_1} \Phi^*_{\alpha_0}. \tag{13}$$

Equations (13) lead to some co-cycles of the type (11), i.e.,

$$\tilde{\nu}_{\alpha_2} = Z^{\alpha_1}{}_{\alpha_2} \tilde{\eta}^*_{\alpha_1} + \tfrac{1}{2} C^{\alpha_0\beta_0}_{\alpha_2} \Phi^*_{\alpha_0} \Phi^*_{\beta_0} \tag{14}$$

that are trivial by virtue of (12). Indeed, from (12) we find

$$Z^{\alpha_1}{}_{\alpha_2} \tilde{\eta}^*_{\alpha_1} = -\tfrac{1}{2} C^{\alpha_0\beta_0}_{\alpha_2} \Phi^*_{\alpha_0} \Phi^*_{\beta_0} \tag{15}$$

hence $\tilde{\nu}_{\alpha_2} \equiv 0$. In consequence, equations (13) do not imply any non-trivial co-cycles at antighost number two, so the antifields $C^*_{\alpha_2}$ are no longer necessary. Thus, formula (13) helps us to derive an irreducible theory. This is the reason for changing the notation δ_R into δ in (13). In order to infer the irreducible gauge transformations corresponding to the irreducible theory we introduce the fields Φ^{α_2} and require that their antifields, denoted by $\Phi^*_{\alpha_2}$, are the non-vanishing solutions to the equations

$$D^{\alpha_2}{}_{\beta_2} \Phi^*_{\alpha_2} = \delta\left(Z^{\alpha_1}{}_{\beta_2} \eta^*_{\alpha_1} + \tfrac{1}{2} C^{\alpha_0\beta_0}_{\beta_2} \Phi^*_{\alpha_0} \Phi^*_{\beta_0}\right). \tag{16}$$

The $\Phi^*_{\alpha_2}$ are fermionic and possess antighost number one. Due to the invertibility of $D^{\alpha_2}{}_{\beta_2}$, the non-vanishing solutions for $\Phi^*_{\alpha_2}$ enforce the irreducibility because equations (16) possess non-vanishing solutions if and only if

$$\delta\left(Z^{\alpha_1}{}_{\beta_2} \eta^*_{\alpha_1} + \tfrac{1}{2} C^{\alpha_0\beta_0}_{\beta_2} \Phi^*_{\alpha_0} \Phi^*_{\beta_0}\right) \neq 0 \tag{17}$$

hence if and only if (11) are not co-cycles. In the meantime, the invertibility of $D^{\alpha_2}{}_{\beta_2}$ emphasizes via (16) that the antifields $\Phi^*_{\alpha_2}$ are δ-exact, which then ensures by virtue of the nilpotency of δ that

$$\delta\Phi^*_{\alpha_2} = 0. \tag{18}$$

With the help of equations (12), (13) and (16) we arrive at

$$\delta\eta^*_{\alpha_1} = Z^{\alpha_0}{}_{\alpha_1} \Phi^*_{\alpha_0} + A_{\alpha_1}{}^{\alpha_2} \Phi^*_{\alpha_2}. \tag{19}$$

By maintaining the definitions from the reducible case

$$\delta\Phi^{\alpha_0} = 0 \qquad \delta\Phi^*_{\alpha_0} = -\frac{\delta S_0}{\delta\Phi^{\alpha_0}} \tag{20}$$

and by setting

$$\delta\Phi^{\alpha_2} = 0 \tag{21}$$

the equations (18)–(21) completely define the irreducible Koszul–Tate complex corresponding to an irreducible theory associated with the original reducible one. At this point we can deduce the action of the irreducible theory $\tilde{S}_0[\Phi^{\alpha_0}, \Phi^{\alpha_2}]$, as well as its gauge invariances. On the one hand, from the standard BRST prescription

$$\delta\Phi^*_{\alpha_2} = -\frac{\delta\tilde{S}_0}{\delta\Phi^{\alpha_2}} \tag{22}$$

compared with (18), we find that

$$\tilde{S}_0[\Phi^{\alpha_0}, \Phi^{\alpha_2}] = S_0[\Phi^{\alpha_0}]. \tag{23}$$

On the other hand, equations (19) lead to the gauge transformations of the irreducible theory in the form

$$\delta_\epsilon\Phi^{\alpha_0} = Z^{\alpha_0}_{\ \ \alpha_1}\epsilon^{\alpha_1} \qquad \delta_\epsilon\Phi^{\alpha_2} = A_{\alpha_1}^{\ \ \alpha_2}\epsilon^{\alpha_1}. \tag{24}$$

Thus, we can conclude that the irreducible theory is based on the original action (see (23)) and the gauge transformations (24). From (23) it is clear that the fields Φ^{α_2} are purely gauge, such that the physical observables of the irreducible system coincide with those of the original reducible theory. The equivalence between the physical observables represents a desirable feature of our irreducible method, which can be gained if we set all the antifields corresponding to the new introduced fields to be δ-closed. On the other hand, as these antifields should not represent non-trivial co-cycles, it is necessary to construct the theory such that they are also δ-exact. Anticipating a bit, we remark that for higher-order reducible theories it is necessary to further enlarge the field and antifield spectra in order to enforce the above-discussed δ-exactness.

2.3. *The case $L = 2$*

In this situation we start with the definitions of δ given by (18)–(21). However, in addition we have to take into account the second-stage reducibility relations (4). On behalf of these supplementary reducibility relations, we find that the matrix $D^{\alpha_2}_{\ \ \beta_2}$ is no longer invertible, as it displays some on-shell null vectors, namely,

$$D^{\alpha_2}_{\ \ \beta_2}Z^{\beta_2}_{\ \ \beta_3} = A_{\beta_1}^{\ \ \alpha_2}C^{\beta_1\beta_0}_{\beta_3}\frac{\delta S_0}{\delta\Phi^{\beta_0}} \approx 0 \tag{25}$$

where the weak equality '$\approx$' means an equality valid when the field equations hold. Thus, in the case of two-stage reducible theories we will consider that $A_{\alpha_1}^{\ \ \alpha_2}$ are chosen such that $\mathrm{rank}(D^{\alpha_2}_{\ \ \beta_2}) \approx M_2 - M_3$. Multiplying (19) by $Z^{\alpha_1}_{\ \ \beta_2}$, we obtain

$$\delta\left(Z^{\alpha_1}_{\ \ \beta_2}\eta^*_{\alpha_1} + \tfrac{1}{2}C^{\alpha_0\beta_0}_{\beta_2}\Phi^*_{\alpha_0}\Phi^*_{\beta_0}\right) = D^{\alpha_2}_{\ \ \beta_2}\Phi^*_{\alpha_2} \tag{26}$$

that together with (25) and (18)–(21) lead to the antighost number two co-cycles

$$\nu_{\alpha_3} = Z^{\alpha_2}_{\ \ \alpha_3}Z^{\alpha_1}_{\ \ \alpha_2}\eta^*_{\alpha_1} + \tfrac{1}{2}C^{\alpha_0\beta_0}_{\alpha_2}Z^{\alpha_2}_{\ \ \alpha_3}\Phi^*_{\alpha_0}\Phi^*_{\beta_0} + A_{\beta_1}^{\ \ \alpha_2}C^{\beta_1\beta_0}_{\alpha_3}\Phi^*_{\alpha_2}\Phi^*_{\beta_0} \tag{27}$$

which are found trivial, $\nu_{\alpha_3} = \delta(-C^{\beta_1\beta_0}_{\alpha_3}\Phi^*_{\beta_0}\eta^*_{\beta_1})$, so there are actually no non-trivial co-cycles at antighost number two. In this way, the only problem that remains to be solved is the δ-exactness of $\Phi^*_{\alpha_2}$, which will further ensure that there are no non-trivial co-cycles at antighost number one. Equations (25) allow us to represent $D^{\alpha_2}_{\ \ \beta_2}$ in the form

$$D^{\alpha_2}_{\ \ \beta_2} = \delta^{\alpha_2}_{\ \ \beta_2} - Z^{\alpha_2}_{\ \ \alpha_3}\bar{D}^{\alpha_3}_{\ \ \beta_3}A_{\beta_2}^{\ \ \beta_3} + A_{\beta_1}^{\ \ \alpha_2}C^{\beta_1\beta_0}_{\alpha_3}\bar{D}^{\alpha_3}_{\ \ \beta_3}A_{\beta_2}^{\ \ \beta_3}\frac{\delta S_0}{\delta\Phi^{\beta_0}} \tag{28}$$

where $\bar{D}^{\alpha_3}_{\beta_3}$ is the inverse of $D^{\alpha_3}_{\beta_3} = Z^{\beta_2}_{\beta_3} A_{\beta_2}^{\alpha_3}$ and $A_{\beta_2}^{\alpha_3}$ are some functions that may involve the fields Φ^{α_0}, chosen to satisfy $\mathrm{rank}(D^{\alpha_3}_{\beta_3}) = M_3$. Inserting (28) in (26) we arrive at

$$\delta\left(Z^{\alpha_1}_{\beta_2}\eta^*_{\alpha_1} + \tfrac{1}{2}C^{\alpha_0\beta_0}_{\beta_2}\Phi^*_{\alpha_0}\Phi^*_{\beta_0} + A_{\beta_1}^{\alpha_2}C^{\beta_1\beta_0}_{\alpha_3}\bar{D}^{\alpha_3}_{\beta_3}A_{\beta_2}^{\beta_3}\Phi^*_{\alpha_2}\Phi^*_{\beta_0}\right) = \Phi^*_{\beta_2} - Z^{\alpha_2}_{\alpha_3}\bar{D}^{\alpha_3}_{\beta_3}A_{\beta_2}^{\beta_3}\Phi^*_{\alpha_2} \tag{29}$$

which show that $\Phi^*_{\beta_2}$ are not δ-exact in the context of the present antifield spectrum. In order to restore the δ-exactness of $\Phi^*_{\beta_2}$ we introduce the bosonic antighost number two antifields $\eta^*_{\alpha_3}$ and define

$$\delta\eta^*_{\alpha_3} = Z^{\alpha_2}_{\alpha_3}\Phi^*_{\alpha_2}. \tag{30}$$

Introducing definitions (30) in equations (29) we deduce that

$$\Phi^*_{\beta_2} = \delta\left(Z^{\alpha_1}_{\beta_2}\eta^*_{\alpha_1} + \tfrac{1}{2}C^{\alpha_0\beta_0}_{\beta_2}\Phi^*_{\alpha_0}\Phi^*_{\beta_0} + A_{\beta_1}^{\alpha_2}C^{\beta_1\beta_0}_{\alpha_3}\bar{D}^{\alpha_3}_{\beta_3}A_{\beta_2}^{\beta_3}\Phi^*_{\alpha_2}\Phi^*_{\beta_0} + \bar{D}^{\alpha_3}_{\beta_3}A_{\beta_2}^{\beta_3}\eta^*_{\alpha_3}\right) \tag{31}$$

which show that $\Phi^*_{\beta_2}$ can be made δ-exact. Replacing (31) in (19) we get that $Z^{\alpha_0}_{\alpha_1}\Phi^*_{\alpha_0}$ are also trivial co-cycles. In conclusion, in the case $L = 2$ the equations (18)–(21) and (30) completely define the irreducible Koszul–Tate complex. Thus, the irreducible theory is also based on action (23), subject to the gauge transformations

$$\delta_\epsilon\Phi^{\alpha_0} = Z^{\alpha_0}_{\alpha_1}\epsilon^{\alpha_1} \qquad \delta_\epsilon\Phi^{\alpha_2} = A_{\alpha_1}^{\alpha_2}\epsilon^{\alpha_1} + Z^{\alpha_2}_{\alpha_3}\epsilon^{\alpha_3} \tag{32}$$

where ϵ^{α_3} are some additional gauge parameters due to (30).

From the above analysis for $L = 1, 2$ it seems that some problems linked with locality appear. Indeed, the matrices $\bar{D}^{\alpha_2}_{\beta_2}$ present in (12) and (13), and also the solutions of equations (16) are, in general, non-local. However, the non-locality involved with (16) compensates, in a certain way, for that from equations (13), such that the irreducible gauge transformations (24) are local. A similar observation can be made with respect to the case $L = 2$. In conclusion, the non-locality present within the intermediate steps of the construction of the irreducible Koszul–Tate complex plays no role in the irreducible theory. Moreover, the non-locality mentioned in the above brings no contribution when comparing the results inferred within the irreducible and reducible procedures (see section 4).

2.4. Generalization to arbitrary L

At this point we can generalize the previous results to an arbitrary L in a simple manner. Acting in a way that ensures on the one hand the nilpotency and acyclicity of the Koszul–Tate differential and on the other hand its irreducibility, we enlarge the field and antifield spectra, and construct the Koszul–Tate complex through

$$\delta\Phi^{\alpha_0} = 0 \qquad \delta\Phi^{\alpha_{2k}} = 0 \qquad k = 1, \ldots, a \tag{33}$$

$$\delta\Phi^*_{\alpha_0} = -\frac{\delta S_0}{\delta\Phi^{\alpha_0}} \qquad \delta\Phi^*_{\alpha_{2k}} = 0 \qquad k = 1, \ldots, a \tag{34}$$

$$\delta\eta^*_{\alpha_{2k+1}} = Z^{\alpha_{2k}}_{\alpha_{2k+1}}\Phi^*_{\alpha_{2k}} + A_{\alpha_{2k+1}}^{\alpha_{2k+2}}\Phi^*_{\alpha_{2k+2}} \qquad k = 0, \ldots, b \tag{35}$$

where the $\Phi^{\alpha_{2k}}$ are bosonic with antighost number zero, the $\Phi^*_{\alpha_{2k}}$ are fermionic, of antighost number one, and the $\eta^*_{\alpha_{2k+1}}$ are bosonic with antighost number two. In the above, the notations a and b mean

$$a = \begin{cases} \dfrac{L}{2} & \text{for } L \text{ even} \\ \dfrac{L+1}{2} & \text{for } L \text{ odd} \end{cases} \qquad b = \begin{cases} \dfrac{L}{2} & \text{for } L \text{ even} \\ \dfrac{L-1}{2} & \text{for } L \text{ odd.} \end{cases} \tag{36}$$

From (33)–(35) we get an irreducible theory described by the action

$$\tilde{S}_0\left[\Phi^{\alpha_0}, \left(\Phi^{\alpha_{2k}}\right)_{k=1,\ldots,a}\right] = S_0\left[\Phi^{\alpha_0}\right] \tag{37}$$

subject to the gauge transformations

$$\delta_\epsilon \Phi^{\alpha_0} = Z^{\alpha_0}_{\ \alpha_1}\epsilon^{\alpha_1} \tag{38}$$

$$\delta_\epsilon \Phi^{\alpha_2} = A_{\alpha_1}^{\ \alpha_2}\epsilon^{\alpha_1} + Z^{\alpha_2}_{\ \alpha_3}\epsilon^{\alpha_3} \tag{39}$$

$$\vdots$$

$$\delta_\epsilon \Phi^{\alpha_{2k}} = A_{\alpha_{2k-1}}^{\ \alpha_{2k}}\epsilon^{\alpha_{2k-1}} + Z^{\alpha_{2k}}_{\ \alpha_{2k+1}}\epsilon^{\alpha_{2k+1}} \tag{40}$$

$$\vdots$$

$$\delta_\epsilon \Phi^{\alpha_{2a}} = \begin{cases} A_{\alpha_{L-1}}^{\ \alpha_L}\epsilon^{\alpha_{L-1}} + Z^{\alpha_L}_{\ \alpha_{L+1}}\epsilon^{\alpha_{L+1}} & \text{for } L \text{ even} \\ A_{\alpha_L}^{\ \alpha_{L+1}}\epsilon^{\alpha_L} & \text{for } L \text{ odd.} \end{cases} \tag{41}$$

The functions $A_{\alpha_{2k-1}}^{\ \alpha_{2k}}$ may depend on the fields Φ^{α_0} and are chosen to satisfy

$$\operatorname{rank}(D^{\beta_k}_{\ \alpha_k}) \approx \sum_{i=k}^{L+1}(-)^{k+i}M_i \qquad k = 1, \ldots, L \tag{42}$$

$$\operatorname{rank}(D^{\beta_{L+1}}_{\ \alpha_{L+1}}) = M_{L+1} \tag{43}$$

where $D^{\beta_k}_{\ \alpha_k} = A_{\alpha_{k-1}}^{\ \beta_k} Z^{\alpha_{k-1}}_{\ \alpha_k}$. We remark that the choice of the functions $A_{\alpha_{k-1}}^{\ \alpha_k}$ is not unique. Moreover, for a definite choice of $A_{\alpha_{k-1}}^{\ \alpha_k}$, equations (42) and (43) are unaffected if we modify the functions $A_{\alpha_{k-1}}^{\ \alpha_k}$ as

$$A_{\alpha_{k-1}}^{\ \alpha_k} \to A_{\alpha_{k-1}}^{\ \alpha_k} + \mu^{\alpha_k}_{\ \beta_{k-2}} Z^{\beta_{k-2}}_{\ \alpha_{k-1}} \tag{44}$$

so that these functions carry some ambiguities. It is known that the reducibility functions $Z^{\alpha_{k-1}}_{\ \alpha_k}$ also display some ambiguities [6]. Throughout the paper we use the conventions $f^{\alpha_k} = 0$ if $k < 0$ or $k > L + 1$.

3. The irreducible BRST symmetry for reducible gauge theories

The derivation of the irreducible Koszul–Tate complex from the above section suggests the possibility to construct an irreducible BRST symmetry associated with the reducible one. This is why, in this section, we point out the derivation of the irreducible BRST symmetry corresponding to the irreducible theory derived within the previous section and show that we can replace the BRST quantization of the original reducible system by that of the irreducible theory. In view of this, we remark that by organizing the fields $(\Phi^{\alpha_0}, \Phi^{\alpha_{2k}})$, as well as the gauge parameters $(\epsilon^{\alpha_1}, \epsilon^{\alpha_{2k+1}})$, into some column vectors Φ^{A_0} and ϵ^{A_1} respectively, the gauge transformations (38)–(41) can be written in a condensed form as $\delta_\epsilon \Phi^{A_0} = Z^{A_0}_{\ A_1}\epsilon^{A_1}$, where $Z^{A_0}_{\ A_1}$ is the appropriate matrix of the gauge generators from (38)–(41) (including $A_{\alpha_{k-1}}^{\ \alpha_k}$ and $Z^{\alpha_{k-1}}_{\ \alpha_k}$). An essential requirement that must be satisfied by the new generators $Z^{A_0}_{\ A_1}$ is their completeness, i.e.,

$$Z^{B_0}_{\ A_1}\frac{\delta Z^{A_0}_{\ B_1}}{\delta \Phi^{B_0}} - Z^{B_0}_{\ B_1}\frac{\delta Z^{A_0}_{\ A_1}}{\delta \Phi^{B_0}} \approx C^{C_1}_{\ A_1 B_1} Z^{A_0}_{\ C_1}. \tag{45}$$

As in general the completeness of the gauge generators depends on the choice of $A_{\alpha_{k-1}}^{\ \alpha_k}$ and also on the reducibility functions of the original theory, in the following we consider only those theories for which (45) hold.

In order to build the irreducible antifield BRST symmetry it is necessary to construct the irreducible Koszul–Tate differential and the irreducible longitudinal exterior derivative along the gauge orbits. The Koszul–Tate differential was constructed in section 2 (see (33)–(35)).

The construction of the longitudinal exterior differential along the gauge orbits, D, follows the general irreducible BRST line [6]. By introducing the minimal ghosts

$$\eta^{A_1} = \begin{pmatrix} \eta^{\alpha_1} \\ \eta^{\alpha_{2k+1}} \end{pmatrix} \tag{46}$$

of pure ghost number one, the definitions

$$\mathrm{D}\Phi^{A_0} = Z^{A_0}_{\ \ A_1}\eta^{A_1} \qquad \mathrm{D}\eta^{A_1} = \tfrac{1}{2}C^{A_1}_{\ \ B_1C_1}\eta^{B_1}\eta^{C_1} \tag{47}$$

together with (45) ensure the weak nilpotency of D without adding any ghosts of ghosts. Under these circumstances, the homological perturbation theory [14–17] guarantees the existence of the irreducible BRST symmetry, s_I.

In what follows we show that it is permissible to substitute the BRST quantization of the reducible theory by that of the irreducible system derived previously. It is obvious that the two theories possess the same classical observables as the fields $(\Phi^{\alpha_{2k}})_{k=1,\ldots,a}$ do not effectively appear in the action of the irreducible system, hence they are purely gauge variables. In consequence, the observables of the irreducible theory actually do not depend on the newly added fields, therefore they satisfy the equations $\frac{\delta F}{\delta\Phi^{\alpha_{2k}}} \approx 0$. Thus, the observables corresponding to the irreducible system, F, involve only the fields Φ^{α_0} and should satisfy just the equations $\frac{\delta F}{\delta\Phi^{\alpha_0}} Z^{\alpha_0}_{\ \ \alpha_1} \approx 0$, which are merely the equations that must be checked by the observables of the reducible theory. As the observables of the irreducible and reducible theories coincide, it follows that the zeroth-order cohomological groups of the irreducible and reducible BRST operators are isomorphic, $H^0(s_I) = H^0(s_R)$. Thus, the irreducible and reducible theories are equivalent from the BRST point of view, i.e., from the point of view of the fundamental equations underlying this formalism, $s^2 = 0$, $H^0(s) = \{\text{physical observables}\}$. All these considerations lead to the conclusion that we can replace the BRST quantization of the reducible theory by that of the irreducible system derived previously.

With all the above ingredients at hand, the BRST quantization of the irreducible theory goes along the standard manner. If one defines the canonical action of s_I through $s_I F = (F, S_I)$, with (,) the antibracket and S_I the canonical generator of the irreducible BRST symmetry, the nilpotency of s_I is expressed by means of the master equation

$$(S_I, S_I) = 0. \tag{48}$$

The existence of the solution to the master equation is guaranteed via the acyclicity of the Koszul–Tate operator at positive antighost numbers. In order to solve the master equation we take $S_I = \sum_{k=0}^{\infty} \overset{(k)}{S}$, with antigh $\overset{(k)}{S} = k$, gh $\overset{(k)}{S} = 0$ and approach the master equation (48) antighost by the antighost level, at the same time requiring the boundary conditions

$$\overset{(0)}{S} = S_0 \qquad \overset{(1)}{S} = \Phi^*_{\alpha_0} Z^{\alpha_0}_{\ \ \alpha_1}\eta^{\alpha_1} + \sum_{k=1}^{a} \Phi^*_{\alpha_{2k}}\left(A^{\ \ \alpha_{2k}}_{\alpha_{2k-1}}\eta^{\alpha_{2k-1}} + Z^{\alpha_{2k}}_{\ \ \alpha_{2k+1}}\eta^{\alpha_{2k+1}}\right). \tag{49}$$

The ambiguities signalized at the end of section 2 in connection with the functions $A^{\ \ \alpha_k}_{\alpha_{k-1}}$ and $Z^{\alpha_{k-1}}_{\ \ \alpha_k}$ induce some ambiguities at the level of the solution to the master equation, S_I. The ambiguity in S_I is completely exhausted by the possibility of performing a canonical transformation in the antibracket [6], so the solution is unique up to such a canonical transformation. In order to fix the gauge, we add the non-minimal variables $(B^{\alpha_{2k+1}}, B^*_{\alpha_{2k+1}})$ and $(\bar{\eta}^{\alpha_{2k+1}}, \bar{\eta}^*_{\alpha_{2k+1}})$, with $k = 0, \ldots, b$, such that we obtain the non-minimal solution $\bar{S} = S_I + \sum_{k=0}^{b} \bar{\eta}^*_{\alpha_{2k+1}} B^{\alpha_{2k+1}}$. A class of appropriate gauge-fixing conditions is given by

$$\chi_{\beta_{2k+1}} \equiv Z^{\beta_{2k}}_{\ \ \beta_{2k+1}} f_{\beta_{2k}}(\Phi^{\beta_{2k}}) + A^{\ \ \beta_{2k+2}}_{\beta_{2k+1}} g_{\beta_{2k+2}}(\Phi^{\beta_{2k+2}}) = 0 \tag{50}$$

where

$$f_{\beta_{2k}}(\Phi^{\beta_{2k}}) \neq Z^{\beta_{2k-1}}_{\beta_{2k}}\rho_{\beta_{2k-1}} \tag{51}$$

$$g_{\beta_{2k+2}}(\Phi^{\beta_{2k+2}}) \neq A^{\beta_{2k+3}}_{\beta_{2k+2}}\gamma_{\beta_{2k+3}}(\Phi^{\beta_{2k+2}}) \tag{52}$$

for some functions $\rho_{\beta_{2k-1}}$ and $\gamma_{\beta_{2k+3}}$. On account of (51) and (52) it is easy to show that the gauge conditions (50) are irreducible even if $A^{\beta_{k+1}}_{\beta_k}A^{\beta_{k+2}}_{\beta_{k+1}} \approx 0$. Thus, a possible class of gauge-fixing fermions can be written as

$$\psi = \sum_{k=0}^{b} \bar{\eta}^{\beta_{2k+1}}\chi_{\beta_{2k+1}} \tag{53}$$

with $\chi_{\beta_{2k+1}}$ given in (50). Eliminating the antifields from S with the help of (53), we deduce the gauge-fixed action, S_ψ, in the standard manner. The gauge-fixing fermion (53) involves (Dirac) δ-functions from the gauge conditions. It is understood that one can shift the gauge conditions by $B_{\beta_{2k+1}}$ in order to reach some Gaussian average representations. Because the gauge conditions are irreducible, the gauge-fixed action displays no residual gauge invariances with respect to the non-minimal sector. Of course, one is free to take any consistent irreducible gauge conditions instead of (50). In conclusion, the path integral of the original reducible theory, quantized accordingly our irreducible procedure, reads as

$$Z_\psi = \int \mathrm{D}\Phi^{A_0}\mathrm{D}\eta^{A_1}\mathrm{D}\bar{\eta}^{A_1}\mathrm{D}B^{A_1}\exp\mathrm{i}S_\psi. \tag{54}$$

Once again we remark that our procedure does not involve ghosts for ghosts, i.e., (54) contains only ghost number one ghost fields. This completes the description of our irreducible treatment for reducible gauge theories.

At this stage, we can emphasize in a clearer manner the role of the newly added fields, $\Phi^{\alpha_{2k}}$, with $k > 0$. In our formalism these fields play a double role, namely, (i) they implement the irreducibility through the gauge transformations (39)–(41), and (ii) they are involved with the irreducible gauge-fixing procedure. In this light, these fields are more relevant than the corresponding non-minimal ones appearing during the gauge-fixing process from the reducible case because, while the newly introduced fields prevent the appearance of the reducibility, the non-minimal fields (in the reducible situation) are mainly an effect of the reducibility, and, consequently, are more passive.

4. Example: the Freedman–Townsend model

Let us apply the prior investigated irreducible approach in the case of the Freedman–Townsend model. We start with the Lagrangian action of the non-Abelian Freedman–Townsend theory [18]

$$S_0^L[B^a_{\mu\nu}, A^a_\mu] = \tfrac{1}{2}\int \mathrm{d}^4x\,(-B^{\mu\nu}_a F^a_{\mu\nu} + A^a_\mu A^\mu_a) \tag{55}$$

where $B^a_{\mu\nu}$ stands for an antisymmetric tensor field, and the field strength, $F^a_{\mu\nu}$, is defined by $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - f^a{}_{bc}A^b_\mu A^c_\nu$. Action (55) is invariant under the first-stage on-shell reducible gauge transformations

$$\delta_\epsilon B^a_{\mu\nu} = \varepsilon_{\mu\nu\lambda\rho}(D^\lambda)^a_b\epsilon^{\rho b} \qquad \delta_\epsilon A^a_\mu = 0 \tag{56}$$

with $(D^\lambda)^a{}_b = \delta^a{}_b\partial^\lambda + f^a{}_{bc}A^{\lambda c}$. The field equations deriving from (55) are

$$\frac{\delta S_0^L}{\delta B^a_{\mu\nu}} \equiv -\frac{1}{2}F^{\mu\nu}_a = 0 \qquad \frac{\delta S_0^L}{\delta A^\mu_a} \equiv A^a_\mu + (D^\lambda)^a{}_b B^b_{\lambda\mu} = 0. \tag{57}$$

The non-vanishing gauge generators of (56), $(Z_{\mu\nu\rho})^{a}{}_{b} = \varepsilon_{\mu\nu\lambda\rho}(D^{\lambda})^{a}{}_{b}$, admit the first-order on-shell reducibility relations

$$(Z_{\mu\nu\rho})^{a}{}_{b}(Z^{\rho})^{b}{}_{c} = -\varepsilon_{\mu\nu\lambda\rho} f^{a}{}_{cd} \frac{\delta S_0^L}{\delta B_{\lambda\rho d}} \tag{58}$$

where the first-stage reducibility functions are expressed by $(Z^{\rho})^{b}{}_{c} = (D^{\rho})^{b}{}_{c}$.

The equivalencies between the general background exposed above and the model under consideration read as $\Phi^{\alpha_0} \leftrightarrow B^a_{\mu\nu}$, $\epsilon^{\alpha_1} \leftrightarrow \epsilon^{\rho b}$, $Z^{\alpha_0}_{\alpha_1} \leftrightarrow \varepsilon_{\mu\nu\lambda\rho}(D^{\lambda})^{a}{}_{b}$, $Z^{\alpha_1}_{\alpha_2} \leftrightarrow (D^{\rho})^{b}{}_{c}$, such that $\alpha_0 \leftrightarrow (a, \mu\nu)$, $\alpha_1 \leftrightarrow (b, \rho)$, $\alpha_2 \leftrightarrow c$. The fields A^a_μ were omitted as their gauge transformations identically vanish, hence they do not contribute to the irreducible treatment. In agreement with our construction, we introduce the fields Φ^{α_2}, which in this case are some scalar fields φ^a. We take $A_{\alpha_1}{}^{\alpha_2}$ to be $(D_\mu)^{a}{}_{b}$, hence the concrete form of (39) is

$$\delta_\epsilon \varphi^a = (D_\mu)^{a}{}_{b}\epsilon^{\mu b}. \tag{59}$$

It is easy to see that the new gauge transformations, namely, (56) and (59) form a complete set, the gauge algebra remaining Abelian. This guarantees the possibility of an appropriate construction of the longitudinal differential along the gauge orbits. With these elements at hand, we pass to the derivation of the gauge-fixed action of the irreducible system associated with the Freedman–Townsend model. The minimal ghost spectrum contains the fermionic ghost number one ghosts $\eta^{\alpha_1} = (\eta^{\mu b})$, while the minimal antifield spectrum is organized as $\Phi^*_{\alpha_0} = (B^{*\mu\nu}_a, \varphi^*_a)$ and $\eta^*_{\alpha_1} = (\eta^*_{\rho b})$, the former antifields having antighost number one and Grassmann parity one, while the latter possess antighost number two and Grassmann parity zero. With the above spectra at hand, the concrete form of the minimal solution to the master equation is

$$S_{\min} = S_0^L + \int \mathrm{d}^4x\, (B^{*\mu\nu}_a \varepsilon_{\mu\nu\lambda\rho}(D^{\lambda})^{a}{}_{b}\eta^{\rho b} + \varphi^*_a (D_\mu)^{a}{}_{b}\eta^{\mu b}). \tag{60}$$

Next, we focus on the gauge-fixing process. We take the gauge conditions as in (50), i.e.,

$$\chi_{\rho b} \equiv -\tfrac{1}{2}\varepsilon_{\mu\nu\lambda\rho}(D^{\lambda})^{c}{}_{b}B^{\mu\nu}_c + (D_\rho)^{c}{}_{b}\varphi_c = 0. \tag{61}$$

The prior gauge conditions are irreducible and are enforced via a non-minimal sector of the type $(\bar{\eta}_{\mu a}, b^{\mu a})$ plus the corresponding antifields. The ghost numbers (gh) and Grassmann parities (ϵ) of the non-minimal fields read as gh $(\bar{\eta}_{\mu a}) = -1$, $\epsilon(\bar{\eta}_{\mu a}) = 1$, gh $(b^{\mu a}) = 0$, $\epsilon(b^{\mu a}) = 0$. The features of their antifields follow from the standard BRST rules. Thus, the non-minimal solution to the master equation is expressed through $S = S_{\min} + \int \mathrm{d}^4x\, \bar{\eta}^*_{\mu a} b^{\mu a}$. Taking the gauge-fixing fermion $\psi = \int \mathrm{d}^4x\, \chi_{\rho b}\bar{\eta}^{\rho b}$, and eliminating the antifields in the standard manner, we arrive at the gauge-fixed action

$$S_\psi = S_0^L + \int \mathrm{d}^4x\, (-\tfrac{1}{2}((D_{[\lambda})^{c}{}_{a}\bar{\eta}_{\rho]c})(D^{[\lambda})^{a}{}_{b}\eta^{\rho]b} - ((D^{\rho})^{c}{}_{a}\bar{\eta}_{\rho c})((D_\mu)^{a}{}_{b}\eta^{\mu b}) + \chi_{\rho b}b^{\rho b}) \tag{62}$$

with $\chi_{\rho b}$ given by (61). The symbol $[\lambda\rho]$ signifies the antisymmetry with respect to the Lorentz indices between brackets. We remark that the gauge-fixed action resulting from our irreducible procedure is Lorentz covariant.

Let us now investigate an Abelian version of Freedman–Townsend theory that is second-stage reducible. We start with the action

$$S_0^L[B^{\mu\nu\rho}_a, A^a_\mu] = \frac{1}{2}\int \mathrm{d}^5x\, \left(\frac{1}{3!}\varepsilon_{\mu\nu\lambda\rho\sigma}B^{\mu\nu\lambda}_a F^{\rho\sigma a} + A^a_\mu A^\mu_a\right) \tag{63}$$

where $B^{\mu\nu\rho}_a$ stand for an Abelian three-form, while $F^{\rho\sigma a}$ is now Abelian, i.e., $F^{\rho\sigma a} = \partial^{[\rho}A^{\sigma]a}$. The gauge transformations of (63) are

$$\delta_\epsilon B^{\mu\nu\rho}_a = \partial^{[\mu}\epsilon^{\nu\rho]}_a \qquad \delta_\epsilon A^a_\mu = 0 \tag{64}$$

and are off-shell second-stage reducible, with the first and second stage reducibility functions respectively given by

$$Z^{\alpha\beta a}_{\gamma b} = \delta_b{}^a \partial^{[\alpha}\delta^{\beta]}_{\gamma} \qquad Z^{\gamma b}_{c} = \delta_c{}^b \partial^{\gamma}. \tag{65}$$

According to the general theory, we add the fields B^{μ}_a that play the role of Φ^{α_2}, the gauge parameters ϵ_a, which are the analogues of ϵ^{α_3}, and take $A_{\alpha_1}{}^{\alpha_2}$ to be the transposed of $Z^{\alpha\beta a}_{\gamma b}$. Therefore, the gauge transformations of B^{μ}_a take the form (see (39))

$$\delta_\epsilon B^{\mu}_a = 2\partial_\nu \epsilon^{\nu\mu}_a + \partial^\mu \epsilon_a. \tag{66}$$

The non-minimal solution to the master equation is expressed by

$$S = S^L_0[B^{\mu\nu\rho}_a, A^a_\mu] + \int \mathrm{d}^5x\,(B^{*a}_{\mu\nu\rho}\partial^{[\mu}\eta^{\nu\rho]}_a + B^{*a}_{\mu}(2\partial_\nu\eta^{\nu\mu}_a + \partial^\mu\eta_a) + \bar{\eta}^{*\mu\nu}_a b^a_{\mu\nu} + \bar{\eta}^*_a b^a). \tag{67}$$

In the last formula, $(\eta^{\mu\nu}_a, \eta_a)$ denote the fermionic ghost number one ghosts, $(B^{*a}_{\mu\nu\rho}, B^{*a}_{\mu})$ are the fermionic ghost number minus one antifields of the corresponding fields, while the remaining fields form the non-minimal sector. If we take some gauge conditions as in (50) by means of the gauge-fixing fermion

$$K = \int \mathrm{d}^5x\,(\bar{\eta}^a_{\mu\nu}(\partial_\rho B^{\rho\mu\nu}_a + \tfrac{1}{2}\partial^{[\mu}B^{\nu]}_a) + \bar{\eta}^a\partial_\mu B^\mu_a) \tag{68}$$

we arrive at the gauge-fixed action

$$S_K = S^L_0[B^{\mu\nu\rho}_a, A^a_\mu] + \int \mathrm{d}^5x\,(\bar{\eta}^a_{\mu\nu}\,\Box\,\eta^{\mu\nu}_a + \bar{\eta}^a\,\Box\,\eta_a + b^a_{\mu\nu}(\partial_\rho B^{\rho\mu\nu}_a + \tfrac{1}{2}\partial^{[\mu}B^{\nu]}_a) + b^a\partial_\mu B^\mu_a) \tag{69}$$

where $\Box = \partial_\lambda\partial^\lambda$. It is easy to see that the gauge-fixed action (69) has no residual gauge invariances.

Finally, we make the comparison between our approach and the reducible BRST treatment in the case of the investigated models. The gauge-fixed actions for the former and latter model within the reducible treatment are respectively given by

$$\begin{aligned} S'_{\psi'} = S^L_0[B^a_{\mu\nu}, A^a_\mu] + \int \mathrm{d}^4x\,&(-\tfrac{1}{2}((D_{[\mu})^c{}_a\bar{\eta}_{\nu]c})(D^{[\mu})^a{}_b\eta^{\nu]b} - ((D_\mu)^c{}_a\bar{\eta}^\mu_c)((D^\nu)^a{}_b\eta^b_\nu) \\ &-((D^\mu)^c{}_a\bar{C}_c)(D_\mu)^a{}_bC^b + \tfrac{1}{8}\epsilon^{\mu\nu\lambda\rho}f^a{}_{bc}((D_{[\mu})^d{}_a\bar{\eta}_{\nu]d})((D_{[\lambda})^c{}_e\bar{\eta}^e_{\rho]})C^b \\ &+(-\tfrac{1}{2}\epsilon^{\mu\nu\lambda\rho}(D_\nu)^b{}_aB_{\lambda\rho b} + (D^\mu)^b{}_a\bar{\eta}_b)b^a_\mu) \end{aligned} \tag{70}$$

$$\begin{aligned} S'_{K'} = S^L_0[B^{\mu\nu\rho}_a, A^a_\mu] + \int \mathrm{d}^5x\,&(\bar{\eta}^a_{\mu\nu}\,\Box\,\eta^{\mu\nu}_a + \bar{\eta}^a\,\Box\,\eta_a + b^a_{\mu\nu}(\partial_\rho B^{\rho\mu\nu}_a + \tfrac{1}{2}\partial^{[\mu}\bar{\eta}^{\nu]}_a) \\ &+b^a\partial_\mu\bar{\eta}^\mu_a - \partial_{[\mu}\bar{\eta}'^a_{\nu]}\partial^{[\mu}C^{\nu]}_a - (\partial_\mu\bar{\eta}'^a)(\partial^\mu C_a) - (\partial^\mu\bar{\eta}'^a_\mu)b'_a + (\partial_\mu C^\mu_a)b''^a). \end{aligned} \tag{71}$$

Apart from the spectra in the irreducible setting, in (70) there appear the additional variables C^a, that are the bosonic ghost number two ghosts, plus the non-minimal fields $(\bar{C}_a, \bar{\eta}_a)$. Along the same line, but with respect to the gauge-fixed action (71), the fields C^μ_a and C_a stand for the ghost number two and three ghosts, respectively, while $(\bar{\eta}^\mu_a, \bar{\eta}'^a_\mu, \bar{\eta}'^a, b'_a, b''^a)$ belong to the non-minimal sector. By performing the identifications

$$\bar{\eta}^a \leftrightarrow \varphi^a \tag{72}$$

respectively,

$$\bar{\eta}^\mu_a \leftrightarrow B^\mu_a \tag{73}$$

between the variables involved with the gauge-fixed actions derived within the irreducible and reducible approaches, the difference between the gauge-fixed actions respectively corresponding to the two models are

$$S'_{\psi'} - S_{\psi} = \int \mathrm{d}^4x \, (-((D^{\mu})^c{}_a \bar{C}_c)(D_{\mu})^a{}_b C^b + \tfrac{1}{8}\epsilon^{\mu\nu\lambda\rho} f^a{}_{bc}((D_{[\mu})^d{}_a \bar{\eta}_{\nu]d})((D_{[\lambda})^c{}_e \bar{\eta}^e_{\rho]})C^b) \tag{74}$$

$$S'_{K'} - S_K = \int \mathrm{d}^5x \, (-\partial_{[\mu}\bar{\eta}'^a_{\nu]}\partial^{[\mu}C^{\nu]}_a - (\partial_\mu \bar{\eta}'^a)(\partial^\mu C_a) - (\partial^\mu \bar{\eta}'^a_\mu)b'_a + (\partial_\mu C^\mu_a)b''^a). \tag{75}$$

We remark that the differences between the gauge-fixed actions are proportional to the ghosts of ghost number greater than one, which are some essential compounds of the reducible BRST quantization. Although identified at the level of the gauge-fixed actions, the fields from (72) and (73) play different roles within the two formalisms. More precisely, the presence of the fields φ^a and B^μ_a prevents the reducibility, while the $\bar{\eta}^a$, and $\bar{\eta}^\mu_a$ respectively, represent an effect of the reducibility. In fact, the fields φ^a and B^μ_a are introduced in order to forbid the existence of the zero modes. In consequence, all the ingredients connected with the zero modes, e.g., the ghosts of ghosts or the non-minimal pyramid, are discarded from the irreducible setting. In this light, we suggestively call the fields φ^a and B^μ_a 'antimodes'. This completes our irreducible treatment.

5. Conclusion

To conclude, in this paper we expose an alternative method of quantizing reducible gauge theories without introducing ghosts of ghosts or their antifields. The cornerstone of our approach is given by the derivation of a Koszul–Tate complex underlying an irreducible gauge theory. As the irreducible gauge system possesses the same physical observables as the original reducible theory, it is legitimate to substitute the BRST quantization of the initial reducible system by that of the irreducible one from the point of view of the main equations underlying the BRST formalism. Then, the general line of the antifield BRST quantization for the irreducible theory is elucidated, some possible gauge conditions being outlined. The general approach is finally exemplified in the case of the Freedman–Townsend model.

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